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CONTROL OF MANIPULATORS – AN OVERVIEW

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CONTROL OF MANIPULATORS - AN OVERVIEW

COMMANDE DES MANIPULATEURS: APERÇU

I.H. Mufti

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ABSTRACT

In recent years several new techniques for the control of manipulators have appeared in the literature. They can be categorized as non-adaptive and adaptive. This report reviews techniques based on inverse dynamics, and also those based on model reference and Popov's hyperstability theory.

RÉSUMÉ

Il a été fait état ces récentes années de plusieurs nouvelles techniques de commande des manipulateurs dans la documentation. Ces techniques sont adaptatives et non adaptatives. Le présent rapport donne un aperçu des techniques qui font appel à la dynamique inverse et de celles qui reposent sur des modèles et sur la théorie d'hyperstabilité de Popov.

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SYMBOLS

Symbol	Definition
$\gamma, \dot{\gamma}, \ddot{\gamma}$	Vectors of Joint Angles, Velocities and Accelerations
$\gamma_d, \dot{\gamma}_d, \ddot{\gamma}_d$	Desired Vectors of Joint-Angles, Velocities and Accelerations
γ_o	Positive Constant (Eq. (33))
ε	Error Vector Between the Actual and Desired Trajectories in the Task Oriented Space
η	Variable Used to Define the Popov Integral (Eq. (33))
$\phi_{p_1}, \phi_{p_2}, \phi_u$	Vectors Generated by the Adaptation Mechanism
$j\omega$	Imaginary Part of Complex Variable

Symbol	Definition
$\cdot$	First Derivative With Respect to Time Variable t
$\cdot\cdot$	Second Derivative With Respect to Time Variable t
diag	Diagonal Matrix
$\|\cdot\|$	Euclidean Norm
$\|\cdot\|_1$	ℓ_1 Norm (Eq. (57))
A	Inertia Matrix in Configuration Space
$\hat{A}$	Inertia Matrix in Task Space
$\hat{A}_o$	Nominal Value of the Inertia Matrix $\hat{A}$
d	Subscript for Desired Trajectory
e	Error Vector Between the Actual and Desired Trajectory in Configuration Space
e	Error Between Plant Position Vector and Reference Model Position Vector
$\underline{e}$	Vector $\begin{pmatrix} e \\ \dot{e} \end{pmatrix}$
f	Coriolis and Centrifugal Force Vector in Configuration Space
$\hat{f}$	Coriolis and Centrifugal Force Vector in Task Space
F	Generalized Force Vector
g	Gravity Force Vector in Configuration Space
$\hat{g}$	Gravity Force Vector in Task Space
h	Vector Function of Joint Variables Representing the Position Vector of the Manipulator Tip
h_{p1}, h_{p2}, h_u	Vectors Defined in Eq. (63)
H	Liapunov Function
H(s)	Transfer Matrix
J	Jacobian Matrix of h
K_1, K_2	Velocity and Position Feedback Gain Matrices
K	System Matrix (Eq. (47))

Symbol	Definition
K_{p_1}, K_{p_2}, K_u	Adaptive Feedback and Feedforward Gain Matrices
L	Lagrangian Function
m	Subscript for Referenc Model
n	Number of Rotational Joints
p	Subscript for Plant
p	Generalized Momentum Vector
P,Q	2n x 2n Positive Definite Matrices (Eq. (48))
r	Input to the Linear System
s	Complex Variable
T	Transpose of a Matrix
-T	Inverse of a Matrix Transpose
T	Kinetic Energy
u	Subscript for Reference Input
u_m	Input to the Reference Model
u_p	Input to the First-order Plant
v	Output of the Linear System
V	Potential Energy
V	Liapunov Function
w	Output of the Nonlinear Time Varying System
x	Position Vector of the Manipulator Tip
x	State Variable of the First-Order Plant
$x_d, \dot{x}_d, \ddot{x}_d$	Desired Vectors of Positions, Velocities and Accelerations of Manipulator Tip
x_m	Position Vector of the Reference Model

CONTROL OF MANIPULATORS - AN OVERVIEW

1. INTRODUCTION

An industrial robot is a general purpose manipulator that consists of several rigid links connected together in the form of an open chain, each link being directly coupled to at most two neighboruing links. The robot can be used as a device for generating position of the last link or as a device for exerting forces on the objects being manipulated where by 'position' we mean 'position and orientation' and by 'force' we mean 'force and torque'. The most frequent task encountered in industrial practice is to transfer the tip of the last link of the manipulator from a given initial point to some desired final point in finite time. The point here belongs to a six-dimensional space representing the position of the last link of the manipulator. The control problem is to determine the joint forces that accomplish this task in some reasonable manner.

In order to solve this control problem we require a mathematical model or description of the manipulator. Such a model can be developed by employing a wide variety of methods [Ref. 1-3]. The complete model is complex and highly nonlinear, and the control of a manipulator, therefore, presents an extremely interesting and challenging problem to the control engineers.

The control problem can be approached either from the modern optimal control theory point of view [Ref. 4, 5] or from the classical control theory point of view [Ref. 1, 6]. In optimal control theory a performance criterion, such as minimum time or minimum energy is specified to measure the effectiveness of the control action. For the complete nonlinear model optimal control theory yields the control as a function of time, that is, as an open loop control. It is practically impossible to obtain a closed-form representation of feedback control as concluded in [Ref. 5] where a time-optimal control is investigated for manipulators with three degrees of freedom. Approximate methods based on linearizing the model around a nominal trajectory and control can however be used to obtain a suboptimal feedback control.

The classical control theory is based on stability considerations and simply stated requires that the control be chosen so as to reduce the error between the manipulator tip point and the desired final point to zero as time goes to infinity. As the system is nonlinear, the second method of Liapunov can be used to obtain a simple feedback control [Ref. 6].

The manipulator task can be specified in another way by requiring that the tip of the last link follows a desired trajectory in the six-dimensional space. The trajectory (also called motion) defines the position of the tip as function of time and the object now is to generate the joint forces so that the manipulator executes the given trajectory. This should be compared with the previously mentioned requirement where the trajectory by which the final point is reached is

not of any consequence and is determined as a part of the control procedure. By specifying the problem in this way the control task is broken into two subtasks - (1) the generation of a desired trajectory between two points and (2) the determination of joint forces so that the tip of the manipulator follows the desired trajectory.

In this report we shall restrict ourselves to the study of the second aspect of the control problem. However, we mention that the problem of trajectory generation can be approached by using either the kinematic model [Ref. 1, 7] or by using the dynamic model and applying optimal control theory [Ref. 8].

We assume now that the trajectory of the manipulator tip is given, either as a function prescribed in advance or computed on-line. In the event the trajectory is prescribed in advance we can, using the dynamic model of the manipulator, compute the control (joint forces) as function of time off-line. This programmed control is adequate if there are no perturbations in the initial values of the trajectory. However, due to the ever presence of perturbations a feedback control may be needed to track the given trajectory. Such a control can be obtained by the usual procedure of linearization around the nominal control and trajectory.

If the manipulator is used to perform a task in unknown environments where the trajectory cannot be prescribed in advance, then it becomes necessary to compute the control on-line. For this the "inverse problem" technique [Ref. 9, 10] (also called "computed torque" technique [Ref. 11, 12]) based on recursive Newton-Euler formulation [Ref. 13] or recursive Lagrangian formulation [Ref. 14] can be employed. Needless to say that this technique is equally valid when the trajectory is prescribed in advance.

All the above mentioned methods require a detailed knowledge of the complete dynamic model of the manipulator. In case the model is only partially known some adaptive control schemes based on model reference approach have been proposed. There are three different ways of designing model reference adaptive control systems (MRACS), [Ref. 15]. They are, respectively, based on the use of local parametric optimization theory, Liapunov's second method and the Popov's hyperstability theory. All these three approaches have been investigated for the control of manipulators [Ref. 16-18][1].

The field of manipulators dynamics and control is relatively new and only a few books have been published on this subject. Apart from one monograph [4], where the control procedures based on optimal control theory and for tasks specified in advance are presented in detail, the other control schemes are scattered in the literature. The purpose of this report is to present briefly some of the newer techniques, non-adaptive as well as adaptive, under one cover. The

[1] One further method of designing MRACS is based on the theory of variable structure systems and sliding modes [Ref. 19, 20].

emphasis will be on reviewing the basic concepts underlying these methods rather than on any implementation details. The report is divided as follows. In Section 2, we present some mathematical preliminaries on the dynamics of manipulators with n rotational degrees of freedom. Section 3 discusses non-adaptive methods based on the inverse problem technique, both in the configuration space (joint-variables) and task oriented space (hand's coordinates). In Section 4, we present adaptive methods based on hyperstability theory. We have chosen this approach over the other two approaches since we believe that hyperstability approach covers the Liapunov's second method approach and no special effort is required to ensure the stability of the overall system as is required in the parametric optimization approach.

2. DYNAMICS OF MANIPULATORS

As a vehicle to present the techniques for the control of manipulators we consider the case of a manipulator with n revolute joints. Its equations of motion can be formulated in many ways [Ref. 1-3]. From structural point of view the Lagrangian formulation is preferred. Let T denote the kinetic energy and V the potential energy. Then the equations of motion are given by the formula:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\gamma}}\right) - \frac{\partial L}{\partial \gamma} = \tau \tag{1}$$

where the Lagrangian function $L(\gamma,\dot{\gamma}) = T-V$, and γ and τ respectively are the n-vectors of joint angles and joint torques. If T is represented as a quadratic form $\frac{1}{2}\dot{\gamma}^T A(\gamma)\dot{\gamma}$ where $A(\gamma)$ is a positive definite matrix, Eq. (1) can be written as a first order system

$$\begin{aligned} \dot{\gamma} &= A^{-1}(\gamma)p \\ \dot{p} &= \frac{\partial L}{\partial \gamma} + \tau = \frac{\partial T}{\partial \gamma} - \frac{\partial V}{\partial \gamma} + \tau \end{aligned} \tag{2}$$

or as a second order system

$$A(\gamma)\ddot{\gamma} + f(\gamma,\dot{\gamma}) + g(\gamma) = \tau \tag{3}$$

where $f(\gamma,\dot{\gamma})$ represents the Coriolis and centrifugal force, $g(\gamma)$ represents the gravitational force and are given by:

$$\begin{aligned} f(\gamma,\dot{\gamma}) &= \dot{A}(\gamma)\dot{\gamma} - \frac{1}{2}\dot{\gamma}^T \frac{\partial A}{\partial \gamma}\dot{\gamma} \\ g(\gamma) &= \frac{\partial V}{\partial \gamma} \end{aligned} \tag{4}$$

The space of joint angles γ is called the configuration space.

The task that a manipulator has to perform is specified in terms of the position and orientation of the tip or hand of the manipulator. Let x denote a 6-vector representing the position and orientation of the hand. The 6-dimensional space to which x belongs is called the task space. A vector γ in the configuration space can be represented by a single vector x in the task space. Let this relation be represented as

$$x = h(\gamma) \tag{5}$$

The vector function h is a complicated function involving sines and cosines of the joint angles. Differentiating twice with respect to time we obtain the relationships between the velocities and accelerations:

$$\dot{x} = \frac{\partial h}{\partial \gamma} \dot{\gamma} = J\dot{\gamma} \tag{6}$$

$$\ddot{x} = J\ddot{\gamma} + \dot{J}\dot{\gamma} \tag{7}$$

The matrix J is a 6×n matrix and is called the Jacobian matrix of h. From Eqs. (6) and (7) $\dot{\gamma}$ and $\ddot{\gamma}$ can be determined uniquely only when J is nonsingular. We consider this case for simplicity and therefore let n=6. Solving Eqs. (6) and (7) for joint velocities and accelerations we can write

$$\dot{\gamma} = J^{-1} \dot{x} \tag{8}$$

$$\ddot{\gamma} = J^{-1}(\ddot{x} - \dot{J}\dot{\gamma}) \tag{9}$$

It should be noted that the Jacobian matrix J is a function of the joint angles γ. Substituting Eq. (9) into Eq. (3) and omitting the argument γ from the matrix $A(\gamma)$ we get:

$$A\, J^{-1}(\ddot{x} - \dot{J}\dot{\gamma}) + f(\gamma, \dot{\gamma}) + g(\gamma) = \tau \tag{10}$$

3. INVERSE PROBLEM TECHNIQUE

We are now in a position to discuss the various schemes for the control of manipulators. We first consider the control in configuration space.

3.1 Control in Configuration Space

Let $\gamma_d(t)$ denote the desired trajectory in configuration space. By differentiating twice with regard to time we can obtain the joint rates $\dot{\gamma}_d(t)$ and joint acceleration $\ddot{\gamma}_d(t)$. Let e(t) denote the error vector between the actual $\gamma(t)$ and the desired $\gamma_d(t)$. We wish to determine a control such that the error vector $e(t) = \gamma(t) - \gamma_d(t)$ tends to zero as $t \to \infty$. For this we assume that e(t) satisfies

$$\ddot{e} + K_1\dot{e} + K_2 e = 0 \tag{11}$$

where K_1 and K_2 are constant diagonal matrices with positive entries. The matrices K_1 and K_2 are called the velocity feedback gain matrix and position feedback gain matrix respectively, and can, in general, be taken as positive definite matrices. Substituting the value of e into Eq. (11) and solving for $\ddot{\gamma}$ we obtain

$$\ddot{\gamma} = \ddot{\gamma}_d - K_1(\dot{\gamma} - \dot{\gamma}_d) - K_2(\gamma - \gamma_d) \tag{12}$$

Assuming that the manipulator is adequately represented by its model Eq. (3), the joint torque vector τ is given by:

$$\tau = A[\ddot{\gamma}_d - K_1(\dot{\gamma} - \dot{\gamma}_d) - K_2(\gamma - \gamma_d)] + f(\gamma,\dot{\gamma}) + g(\gamma) \tag{13}$$

The control in Eq. (13) contains the acceleration compensation term $A\ddot{\gamma}_d$, the compensation term for Coriolis and centrifugal force $f(\gamma,\dot{\gamma})$, the gravity compensation term $g(\gamma)$ and the feedback term $K_v\dot{e} + K_p e$ where $K_v = A(\gamma)K_1$ and $K_p = A(\gamma)K_2$. It should be noted that the matrices K_v and K_p are variable by virtue of the configuration dependent inertia matrix $A(\gamma)$.

For real-time control an efficient algorithm is needed to compute the joint torques from Eq. (13). This is provided by the Newton-Euler formulation [Ref. 13] where given the joint angles, rates and accelerations (inverse problem), the joint torques are computed by first transforming the velocities and accelerations from the base to the tip of the manipulator link by link and then the forces are transformed back from the tip to the base. In the case of Eq. (13) the joint angles γ and the joint rates $\dot{\gamma}$ are measured and the accelerations are computed by

$$\ddot{\gamma} = \ddot{\gamma}_d - K_1(\dot{\gamma} - \dot{\gamma}_d) - K_2(\gamma - \gamma_d)$$

Another general but perhaps less efficient procedure for real-time control is the recursive Lagrangian formulation [Ref. 14]. A method [Ref. 10] based on tabulating the configuration dependent terms has been also tried. A comparison of computational complexities of these methods is presented in [Ref. 14].

For the special case where γ_d = constant, Eq. (13) becomes

$$\tau = -A[K_1\dot{\gamma} + K_2(\gamma - \gamma_d)] + f(\gamma,\dot{\gamma}) + g(\gamma) \tag{14}$$

This case corresponds to that of moving the tip of the manipulator from some initial configuration to the final configuration γ_d.

Remark 1. For this special case a simple controller with constant gain matrices and not containing the Coriolis and centrifugal force compensation term $f(\gamma,\dot{\gamma})$ is given in [Ref. 6]. In fact, by constructing

a Liapunov function H,

$H(\gamma-\gamma_d,\dot{\gamma}) = p^T\dot{\gamma} - L - V(\gamma_d) - g^T(\gamma_d)(\gamma-\gamma_d) + \frac{1}{2}(\gamma-\gamma_d)^T K_2(\gamma-\gamma_d)$

it is shown that every solution of Eq. (2) tends to the equilibrium position $(\gamma_d,0)$ if

$$\tau = -K_1\dot{\gamma} - K_2(\gamma-\gamma_d) + g(\gamma_d) \tag{15}$$

where K_1 and K_2 are positive definite matrices with K_2 satisfying $K_2 + \frac{\partial^2 V}{\partial \gamma^2} > 0$.

Remark 2. Eq. (11) is usually taken for the error equation. This represents proportional plus derivative control. More generally, we can write the error equation as

$$\ddot{e} = u$$

with the control u determined in such a way that the error tends to zero as $t\to\infty$ (stability problem) or as $t\to t_f$ (terminal control problem) where t_f is finite. These ideas are pursued in [Ref. 21] where proportional + integral + derivative (PID) control and time optimal control are investigated.

Remark 3. In the present day practice, the manipulator trajectory control is primarily realized by local independent servo systems (one for each joint) [Ref. 1]. For the set point regulation PID (or PD) control of the form $\tau = -K_1(\gamma-\gamma_d) - K_i z - K_2\dot{\gamma}$ is chosen where $\dot{z} = (\gamma-\gamma_d)$, K_1 K_i and K_2 are positive diagonal matrices determined on the basis of some simple dynamics (assuming A to be a constant diagonal matrix, neglecting Coriolis and centrifugal forces) or by linearization around the set point. The integral term is included because of its tracking quality in the face of disturbances.

3.2 Control in Task Space

The trajectories are naturally prescribed in the task space. To use the results described in Section 3.1 we must transform or resolve the desired trajectory $x_d(t)$ in the task space to $\gamma_d(t)$. In conventional methods this approach has been taken where γ_d, computed by solving the nonlinear Eq. (5) or by integrating Eq. (18) (resolved motion rate control [Ref. 22]) is fed to the joint servos. Here we shall discuss a procedure [Ref. 23,24] which controls the manipulator directly in the task-oriented coordinates.

As in Eq. (11) we write the error equation as

$$\ddot{\varepsilon} + K_1\dot{\varepsilon} + K_2\varepsilon = 0 \tag{16}$$

where $\varepsilon = x(t) - x_d(t)$. Solving Eq. (16) for $\ddot{x}$ and substituting it into Eq. (10) we obtain

$$\tau = AJ^{-1}[\ddot{x}_d - K_1(\dot{x}-\dot{x}_d) - K_2(x-x_d) - \dot{J}\dot{\gamma}] + f(\gamma,\dot{\gamma}) + g(\gamma) \qquad (17)$$

which can be computed efficiently by using the recursive Newton-Euler formultion with γ, $\dot{\gamma}$ measured and $\ddot{\gamma}$ given by:

$$\ddot{\gamma} = J^{-1}[\ddot{x}_d - K_1(\dot{x}-\dot{x}_d) - K_2(x-x_d) - \dot{J}\dot{\gamma}] \qquad (18)$$

In Eq. (18) x and $\dot{x}$ are computed using Eqs. (5) and (6) respectively.

For the special case when x_d = constant, the joint torque τ becomes

$$\tau = -AJ^{-1}[K_1\dot{x} + K_2(x-x_d) + \dot{J}\dot{\gamma}] + f(\gamma,\dot{\gamma}) + g(\gamma) \qquad (19)$$

As in the case of point to point control in configuration space, a simple controller can be found. In fact, by constructing a suitable Liapunov function it can be shown [Ref. 6] that the manipulator can be stabilized if

$$\tau = -J^T[K_1\dot{x} + K_2(x-x_d)] + g(\gamma) \qquad (20)$$

where K_1 and K_2, as before, are positive definite matrices. It should be noticed that the torque τ in Eq. (20) does not require the computation of the inverse of the Jacobian matrix J which, in general, is quite troublesome. Moreover, no compensation for the Coriolis and centrifugal term $f(\gamma,\dot{\gamma})$ is requried.

Remark 4. It is not our intention to consider the implementation problem of the control schemes discussed in this report. However we note that for the Newton-Euler formulation with joint angles, rates and accelerations given, it takes on the average 4.5 ms to compute the joint forces on a PDP 11/45 using a program written in floating point assembly language for the Stanford manipulator [Ref. 13] while the execution time for a Fortran program is 33.5 ms. For a Puma robot arm, the computation time for the joint torques is within 3ms if complex trigonometric functions are implemented as a table lookup [Ref. 25]. The use of multi-processor system has been also studied. It is shown [Ref. 26] that by assigning one processor to each joint and by scheduling the tasks in an optimal way the execution time for a Fortran program is 10.7 ms. For computing the execution time it is assumed that the execution time for a multiplication is 50 μs and that for an addition is 40 μs.

4. MODEL REFERENCE ADAPTIVE CONTROL

The control techniques presented in the previous section are based on the assumption that a detailed model of the manipulator is available. However, due to the uncertainty and the variations in

manipulator's parameters such as that occur when the manipulator grips an unknown object, this assumption does not hold and the control laws obtained previously may not be adequate. In these cases what is needed is a control system that eliminates the effects of plant (physical system) parameter variations on the degradation of its performance. Such a control system is called an adaptive control system. There are several ways of obtaining an adaptive control. Here we only consider one particular form of an adaptive control called model reference adaptive control (MRAC). Basically, the control system designed by this approach includes a reference model which specifies the desired performance for the plant, and an adaptation mechanism to generate the input signal to the plant so that the error between the states or outputs of the plant and the reference model tends to zero as $t \to \infty$.

For linear systems there are two popular MRAC design methods, one based on Liapunov's second method and the other on hyperstability theory [Ref. 15, 27]. In this section we shall discuss only those design methods that are based on hyperstability theory. However before applying these methods to the control of manipulators which are nonlienar, it is instructive to consider a simple example of a first order constant linear system.

Example. Consider a plant given by

$$\dot{x} + a_p x = b_p u_p, \quad b_p > 0 \tag{21}$$

and the reference model described by

$$\dot{y} + a_m y = b_m u_m, \quad a_m, b_m > 0 \tag{22}$$

The parameters a_p and b_p of the plant are unknown. Let the error e be defined by

$$e = x - y \tag{23}$$

The problem is to determine the input signal u_p such that for any initial value of e, $e(t) \to 0$ as $t \to \infty$. Using Eqs. (21)-(23) the error equation is given by

$$\dot{e} = -a_m e + (a_m - a_p)x + (b_p u_p - b_m u_m) \tag{24}$$

If the parameters a_p and b_p were known then by setting the last two expressions on the right hand side of Eq. (24) to zero we obtain

$$u_p = -k_p^o x + k_u^o u_m \tag{25}$$

where $k_p^o = (a_m - a_p)/b_p$ and $k_u^o = b_m/b_p$. With this value of u_p the error Eq. (24) reduces to $\dot{e} = -a_m e$ and hence $e(t) \to 0$ as $t \to \infty$ for any initial value

e(o). For the case under consideration where a_p and b_p are not known it is reasonable to assume that the input signal u_p to the plant be taken as:

$$u_p = -k_p x + k_u u_m \tag{26}$$

where k_p and k_u are some functions of time. Substituting Eq. (26) into Eq. (24) the error equation can be written as:

$$\dot{e} = -a_m e + b_p(k_p^o - k_p)x + b_p(k_u - k_u^o)u_m \tag{27}$$

The problem now is to determine the adaptive gains k_p and k_u such that $e(t) \to o$ as $t \to \infty$ for any initital values e(o), $k_p(o)$ and $k_u(o)$. As the system described by Eq. (27) is time-varying nonlinear system this design objective can be achieved by constructing a suitable Liapunov function.

Let us write $\delta_1 = k_p^o - k_p$ and $\delta_2 = k_u - k_u^o$ and assume that x and u_m are bounded. As a candidate consider the following Liapunov function V [Ref. 15, 28, 29]

$$V(e,\delta_1,\delta_2) = \frac{1}{b_p} e^2 + \frac{1}{f_p} \delta_1^2 + \frac{1}{g_u} \delta_2^2 \tag{28}$$

where f_p and g_u are positive constants. The function V is positive definite and in addition $V \to \infty$ as $e^2 + \delta_1^2 + \delta_2^2 \to \infty$. Its total time derivative in view of Eq. (27) is

$$\begin{aligned} \dot{V} &= -\frac{2a_m}{b_p} e^2 + 2e(\delta_1 x + \delta_2 u_m) + \frac{2}{f_p} \delta_1 \dot{\delta}_1 + \frac{2}{g_u} \delta_2 \dot{\delta}_2 \\ &= -\frac{2a_m}{b_p} e^2 + \frac{2\delta_1}{f_p} (\dot{\delta}_1 + f_p ex) + \frac{2\delta_2}{g_u} (\dot{\delta}_2 + g_u e u_m) \end{aligned} \tag{29}$$

To make $\dot{V}$ negative semidefinite we choose

$$\begin{aligned} \dot{\delta}_1 &= -f_p ex \\ \dot{\delta}_2 &= -g_u e u_m \end{aligned} \tag{30}$$

Hence the equilibrium positon (0,0,0) is uniformly stable and e, δ_1 and δ_2 are bounded. Furthermore, using the arguments similar to that in [Ref. 28, 29] it can be established that $e(t) \to 0$ as $t \to \infty$ for any initial

values $e(o)$, $\delta_1(0)$, $\delta_2(0)$, and x is bounded.

In terms of the adaptive gains k_p and k_u, Eq. (30) can be written as

$$\dot{k}_p = f_p\, ex$$
$$\dot{k}_u = -g_u\, eu_m \tag{31}$$

The block diagram of the proposed adaptive system is represented in Fig. 1.

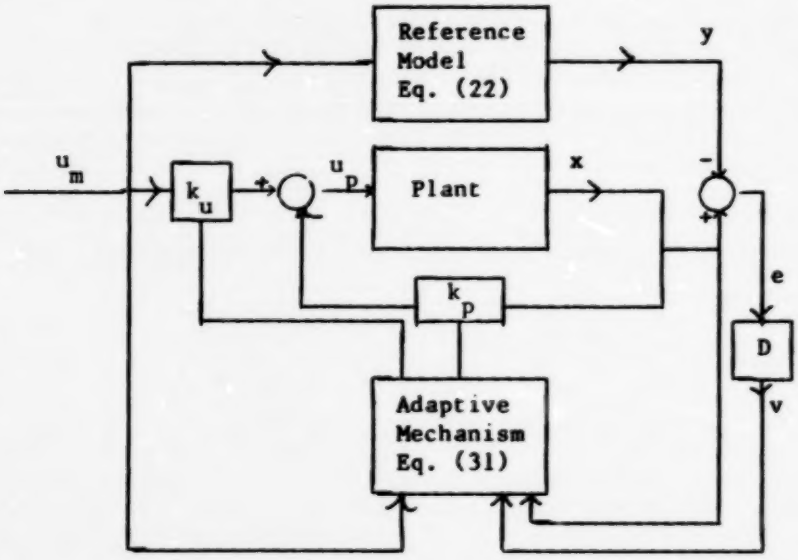

Fig. 1 Block Diagram of MRAC System

The adaptive law given by Eq. (31) is known as an integral adaptation. Other adaptation laws can be obtained by constructing suitable Liapunov functions. However it is not known in general how to widen the class of such functions so as to yield a large family of adaptation laws. This difficulty is somewhat circumvented by the use of hyperstability theory.

To introduce the concepts of hyperstability theory let us rewrite the error Eq. (27) as the following feedback system:

$$\dot{e} = -a_m e + r,\quad v = De$$
$$r = -w = b_p\delta_1 x + b_p\delta_2 u_m \tag{32}$$

with input r, output v and the feedback control w. A block diagram representation is shown in Fig. 2. It consists of two parts, one lienar

and time invariant, and the other nonlinear and time varying. Here D=1 and e,v,w are all scalars. However in the general case D is matrix and

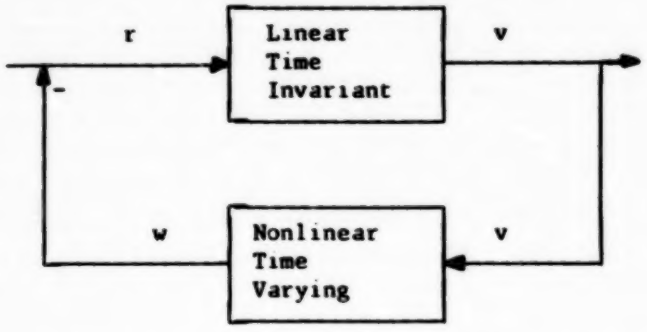

Fig. 2 Block Diagram of Standard Nonlinear Feedback System Used In Hyperstability Theory

e,v,w are vectors of appropriate dimensions.

Definition. The feedback system is said to be asymptotically hyperstable if the state e=0 is globally asymptotically stable for all nonlinear feedback w satisfying the Popov inequality [Ref. 30],

$$\eta(o,t_1) = \int_o^{t_1} v^T w dt \geq -\gamma_o^2 \text{ for all } t_1 \geq 0 \qquad (33)$$

where γ_o^2 is a positive constant independent of t_1.

We recall that the equilibrium state e=0 is called globally asymptotically stable if it is asymptotically stable for all initial values e(o). We also note that, all signals appearing in the feedback control are assumed to be bounded in some sense.

For ensuring asymptotic hyperstability we quote the following result:

Result. The necessary and sufficient condition for the feedback system to be asymptotically hyperstable is that the transfer matrix of the linear system be strictly positive real.

We mention that an mxm matrix H(s) of real rational functions is strictly positive real if

i) all elements of H(s) have no poles in the closed right half plane Re $s \geq 0$, and

ii) the matrix $H(j\omega) + H^T(-j\omega)$ is a positive definite Hermitian matrix for all real ω.

In the case of single input, single ouput system (m=1) the second condition means that $\mathrm{Re}[h(j\omega)] > 0$, $-\infty < \omega < +\infty$.

Assuming that the transfer function or matrix is strictly positive real, the problem of determining adaptive laws reduces to that of determining the class of functions w satisfying the Popov inequality (33). Several general results concerning the solutions to this inequality are known (see, for example, [Ref. 15]). Here we shall consider only some particular solutions and derive them by using the first order plant under consideration.

Let us consider the first term $-b_p \delta_1 x$ in w. The second term can be treated similarly. In order that inequality (33) be satisfied it is sufficient that

$$\int_o^{t_1} v\delta_1 x dt \leq \gamma_{o1}^2 \tag{34}$$

We write k_p as

$$k_p = \int_o^t f_{p_1} vx d\tau + f_{p_2} vx + f_{p3} \tag{35}$$

where f_{p_1} is some function of $(t-\tau)$, f_{p_2} a function of t and f_{p_3} a constant. Substituting this value of k_p into the left hand side of Eq. (34) we obtain:

$$\int_o^{t_1} v\delta_1 x d\tau = \int_o^{t_1} vx(k_p^o - f_{p_3} - \int_o^t f_{p_1} vx d\tau)dt - \int_o^{t_1} f_{p_2} (vx)^2 dt \tag{36}$$

We now show that if f_{p_1} is a positive constant and f_{p_2} a nonnegative constant, there exists a γ_{01}^2 such that inequality (34) is satisfied. Let us write

$$h(t) = k_p^o - f_{p_3} - \int_o^t f_{p_1} vx d\tau$$

Differentiating it with respect to t we have

$$\dot{h}(t) = -f_{p_1} vx$$

Hence the first integral in Eq. (36) can be written as:

$$-\frac{1}{f_{p_1}} \int_o^{t_1} \dot{h}(t)\, h(t) dt = -\frac{1}{2f_{p_1}} h^2(t) \Big|_o^{t_1} = -\frac{1}{2f_{p_1}} (h^2(t_1) - h^2(o))$$

Substituting this into Eq. (36) we obtain

$$\int_o^{t_1} v\delta_{i_1} x d\tau = -\frac{1}{2f_{p_1}}(h^2(t_1) - h^2(o)) - f_{p_2}\int_o^{t_1}(vx)^2 dt$$

$$\leq \frac{1}{2f_{p_1}} h^2(o) = \gamma_{o1}^2$$

This proves that inequality (34) is satisfied with $\gamma_{o1}^2 = \frac{1}{2f_{p_1}} h^2(o)$.

In a similar manner, we can write

$$k_u = -\int_o^t g_{u_1} vx d\tau - g_{u_2} vx + g_{u_3} \tag{37}$$

With f_{pi} and g_{ui}, i=1,2 taken as positive constants, the law given by Eqs. (35) and (37) is called integral + proportional adaptation. We note that with $f_{p_2}=g_{u_2}=0$ the adaptation law reduces to that obtained by the Liapunov's method.

Other adaptation laws can be obtained. Notice that Popov inequality is satsified if $f_{p_2}(t) \geq 0$ and $g_{u_2}(t) \geq 0$. Thus, for example, by taking

$$\begin{aligned} f_{p_2}(t) &= f_{p_2}\frac{1}{|vx|} \text{ or } = f_{p_2}\frac{1}{|v|} \text{ or } = f_{p_2}\frac{1}{|x|}, \ f_{p_2} \geq 0 \\ g_{u_2}(t) &= g_{u_2}\frac{1}{|vx|} \text{ or } = g_{u_2}\frac{1}{|v|} \text{ or } = \frac{1}{|x|}, \ g_{u_2} \geq 0 \end{aligned} \tag{38}$$

we have relay adaptation of the form sgn vx, x sgn v or v sgn x respectively. As far as the functions $f_{p_i}(t-\tau)$ and $g_{u_i}(t-\tau)$ are concerned, according to the general theory they could be any functions provided their Laplace transforms are positive real functions with a pole at s=0.

We now consider the application of MRAC to the control of manipulators with trajectories prescribed in the task space. By multiplying Eq. (10) on the left by J^{-T} the equations of motion of the manipulator in the task-oriented space can be written as:

$$\hat{A}(\gamma)\ddot{x} + \hat{f}(\gamma,\dot{\gamma}) + \hat{g}(\gamma) = F \tag{39}$$

where

$$\begin{aligned} &\hat{A}(\gamma) = J^{-T}AJ^{-1}, \ \hat{f}(\gamma,\dot{\gamma}) = -\hat{A}\dot{J}\dot{\gamma} + J^{-T}f(\gamma,\dot{\gamma}) \\ &\hat{g}(\gamma) = J^{-T}g(\gamma), \ F = J^{-T}\tau \end{aligned} \tag{40}$$

By premultipmying Eq. (39) by $\hat{A}^{-1}$ we obtain

$$\ddot{x} = -\hat{A}^{-1}(\hat{f} + \hat{g} - F) \tag{41}$$

Let the reference model be described by

$$\ddot{x}_m + K_1\dot{x}_m + K_2 x_m = u_m \tag{42}$$

The equation for the error $e = x - x_m$ can be obtained by subtracting Eq. (42) from Eq. (41):

$$\ddot{e} + K_1\dot{e} + K_2 e = -A^{-1}(\hat{f}+\hat{g}-F) + K_1\dot{x} + K_2 x - u_m \tag{43}$$

The problem as before is to determine F such that the error $e \to o$ as $t \to \infty$. If the manipulator model is adequately known then by setting the right-hand side of Eq. (43) to zero we obtain

$$F = \hat{A}(u_m - K_1\dot{x} - K_2 x) + \hat{f} + \hat{g} \tag{44}$$

Using Eq. (40) and setting $u_m = \ddot{x}_d + K_1\dot{x}_d + K_2 x_d$ we get

$$\tau = A\,J^{-1}[\ddot{x}_d - K_1(\dot{x}-\dot{x}_d) - K_2(x-x_d)] - AJ^{-1}\dot{J}\dot{\gamma} + f(\gamma,\dot{\gamma}) + g(\gamma),$$

which is the same expression as that given by Eq. (17).

In order to apply the hyperstability theory to the design of MRAC system we write Eq. (43) in the equivalent nonlinear feedback form:

$$\dot{\underline{e}} = K\,\underline{e} + \begin{pmatrix} 0 \\ I \end{pmatrix} r, \quad v = D\,\underline{e} \tag{45}$$

$$r = -w = -\hat{A}^{-1}(\hat{f}+\hat{g}-F) + K_1\dot{x} + K_2 x - u_m \tag{46}$$

where I is a unit matrix of appropriate dimensions and

$$\underline{e} = \begin{pmatrix} e \\ \dot{e} \end{pmatrix}, \quad K = \begin{pmatrix} 0 & I \\ -K_1 & -K_2 \end{pmatrix} \tag{47}$$

Our first step is to ensure that the transfer matrix $H(s) = D(sI-K)^{-1}\begin{pmatrix} 0 \\ I \end{pmatrix}$ be strictly positive real. Since K is assumed to have roots with negative real parts this positivity condition is satisfied if $D = P_2$ where P_2 is an nx2n matrix formed by the last n rows of a 2nx2n positive definite matrix P satisfying the Liapunov equation

$$K^T P + PK = -Q \tag{48}$$

for a given 2nx2n positive definite matrix Q.

The second step in the application of hyperstability theory is to find a w such that the Popov inequality (33) is satisfied. If $\hat{f}$ and $\hat{g}$ were linear and $\hat{A}$ a constant matrix then, following Eq. (44), we write the adaptation law as

$$F = K_u u_m - K_{p_1}\dot{x} - K_{p_2} x \tag{49}$$

Substituting this value of F and taking $\hat{f} = K_1\dot{x}$, $\hat{g} = K_2 x$ we obtain

$$w = \hat{A}^{-1}(\bar{K}_1\dot{x} + \bar{K}_2 x - K_u u_m + K_{p_1}\dot{x} + K_{p_2} x) - K_1\dot{x} - K_2 x + u_m$$
$$= -\hat{A}^{-1}(K^o_{p_1} - K_{p_1})\dot{x} - \hat{A}^{-1}(K^o_{p_2} - K_{p_2})x - \hat{A}^{-1}(K_u - K^o_u)u_m \qquad (50)$$

where

$$K^o_{p_1} = \hat{A}K_1 - K_1, \quad K^o_{p_2} = \hat{A}K_2 - K_2, \quad K^o_u = \hat{A} \qquad (51)$$

Eq. (50) has the same form as that given in Eq. (32) except that here we have matrices instead of scalar quantities. Following procedures similar to that employed in the scalar case the integral adaptation is given by:

$$\dot{K}_{p_1} = F_{p_1} P_2 \underline{e}\,\dot{x}^T, \quad \dot{K}_{p_2} = F_{p_2} P_2 \underline{e}\, x^T, \quad \dot{K}_u = -G_u P_2 \underline{e}\, u_m^T \qquad (52)$$

where F_{p_1}, F_{p_2} and G_u are nxn constant positive definite matrices. For the proportional adaptation we have

$$K_{p_1} = \bar{F}_{p_1} P_2 \underline{e}\,\dot{x}^T, \quad K_{p_2} = \bar{F}_{p_2} P_2 \underline{e}\, x^T, \quad K_u = -\bar{G}_u P_2 \underline{e}\, u_m^T \qquad (53)$$

where again the matrices $\bar{F}_{p_1}$, $\bar{F}_{p_2}$ and $\bar{G}_u$ are positive definite. In particular they can be chosen as positive constants. Writing v for $P_2\underline{e}$ and substituting Eq. (53) into Eq. (49) the adaptive control law corresonding to the proportional adaptation becomes

$$F = -(\bar{G}_u \|v\|\|u_m\|^2 + \bar{F}_{p_1} \|v\|\|\dot{x}\|^2 + \bar{F}_{p_2} \|v\|\|x\|^2) \qquad (54)$$

where $\|\cdot\|$ denotes the Euclidean norm.

In the case of relay adaptation we can, for example, take the adaptive gains as:

$$K_{p_1} = f_{p_1} \operatorname{sgn} v\, \dot{x}^T, \quad K_{p_2} = f_{p_2} \operatorname{sgn} v\, x^T, \quad K_u = -g_u \operatorname{sgn} v\, u_m^T \qquad (55)$$

or as:

$$K_{p_1} = f_{p_1} \frac{v}{\|v\|} (\operatorname{sgn} \dot{x})^T, K_{p_2} = f_{p_2} \frac{v}{\|v\|} (\operatorname{sgn} x)^T, K_u = -g_u \frac{v}{\|v\|} (\operatorname{sgn} u_m)^T \qquad (56)$$

where $\operatorname{sgn} v = (\operatorname{sgn} v_1, \operatorname{sgn} v_2 \ldots, \operatorname{sgn} v_n)^T$ and f_{p_1}, f_{p_2} and g_u are positive constants. Taking the adaptive gains as in Eq. (56), for example, F becomes:

$$F = -\frac{v}{\|v\|}\left(f_{p_1}\|\dot{x}\|_1 + f_{p_2}\|x\|_1 + g_u\|u_m\|_1\right) \tag{57}$$

where $\|x\|_1 = |x_1| + |x_2| + \ldots + |x_n|$. Notice that F is not defined when $v = 0$.

We now consider the case when $\hat{f}$, $\hat{g}$ are nonlinear and the matrix $\hat{A}$ is configuration dependent. In this case it is not at all obvious what form the adaptive control law F should have. Let us assume that the plant is partially known and the gravity term $\hat{g}$ can be compensated. This is a reasonable assumption since $\hat{g}$ can be computed easily. If we further assume that the nonlinear term $\hat{f}$ is negligibly small and the matrix $\hat{A}$ is constant (or slowly varying) we can use linear theory and set

$$F = \hat{g} + (K_u u_m - K_{p_1}\dot{x} - K_{p_2}x) \tag{58}$$

However, if the above assumption does not hold then the adaptive controller must compensate for the nonlinearity and the variation in the inertia matrix. Thus we modify Eq. (58) and write F as:

$$F=\hat{g}+(K_u u_m - K_{p_1}\dot{x} - K_{p_2}x) - \phi_u(u_m,v) - \phi_{p_1}(\dot{x},v) - \phi_{p_2}(x,v) \tag{59}$$

where ϕ_u, ϕ_{p_1} and ϕ_{p_2} are n-vectors. It should be noted that we could have written F as:

$$F = \hat{g} - \phi_u(u_m,v) - \phi_{p_1}(\dot{x},v) - \phi_{p_2}(x,v) \tag{60}$$

by omitting the term $(K_u u_m - K_{p_1}\dot{x} - K_{p_2}x)$. This type of F has been investigated in [Ref. 18] and leads to an adaptive control exhibiting chattering phenomenon.

In order to obtain an adaptive control having integral + relay type adaptation we substitute Eq. (59) into the expression for w and obtain:

$$w = \hat{A}^{-1}(\hat{f} - K_u u_m + K_{p_1}\dot{x} - K_{p_2}x + \phi_u + \phi_{p_1} + \phi_{p_2}) - K_1\dot{x} - K_2 x + u_m \tag{61}$$

By adding and subtracting the expression $\hat{A}_o^{-1}(K_{p_1}\dot{x} + K_{p_2}x - K_u u_m)$ in Eq. (61), w can be rewritten as:

$$\begin{aligned} w = &-[\hat{A}_o^{-1}(\hat{A}_o K_1 - K_{p_1})\dot{x} + \hat{A}_o^{-1}(\hat{A}_o K_1 - K_{p_2})x + \hat{A}_o^{-1}(K_u - \hat{A}_o)u_m] \\ &-(h_{p_1} - \hat{A}^{-1}\phi_{p_1}) - (h_{p_2} - \hat{A}^{-1}\phi_{p_2}) - (h_u - \hat{A}^{-1}\phi_u) \end{aligned} \tag{62}$$

where $\hat{A}_o$ is a constant, but unknown matrix corresponding to some nominal value of $\hat{A}$ and

$$h_{p_1} = -\hat{A}^{-1}\hat{f} + (\hat{A}_o^{-1} - \hat{A}^{-1})K_{p_1}\dot{x}$$
$$h_{p_2} = (\hat{A}_o^{-1}-\hat{A}^{-1})\,K_{p_2}x,\quad h_u = (\hat{A}^{-1}-\hat{A}_o^{-1})K_u u_m \tag{63}$$

It should be noted that the expression in square brackets in Eq. (62) is similar to the one obtained for the linear system (Eq. (50)). Also h_{p_1}, h_{p_2} and h_u become zero if $\hat{A} = \hat{A}_o$ and $\hat{f} = 0$.

For the integral adaptation law, from Eq. (52) we have:

$$\hat{K}_{p_1} = f_{p_1} v\,\dot{x}^T,\quad \hat{K}_{p_2} = f_{p_2} v\,x^T,\quad \hat{K}_u = -g_u v\,u_m^T \tag{64}$$

where f_{p_1}, f_{p_2} and g_u are positive constants.

We now determine the expressions for ϕ_{p_1}, ϕ_{p_2} and ϕ_u so that the Popov inequality is satisfied. For this it is sufficient that

$$\int_o^{t_1} v^T(h_{p_1}-\hat{A}^{-1}\phi_{p_1})dt \le \gamma_{p_1}^2,\quad \int_o^{t_1} v^T(h_{p_2}-\hat{A}^{-1}\,\phi_{p_2})dt \le \gamma_{p_2}^2$$
$$\int_o^{t_1} v^T(h_u-\hat{A}^{-1}\phi_u)dt \le \gamma_u^2 \tag{65}$$

be satisfied. Consider the first inequality in Eq. (65) as others can be treated similarly. For this to be satisfied it is sufficient that

$$v^T h_{p_1} \le v^T\hat{A}^{-1}\phi_{p_1} \tag{66}$$

Following Eq. (57) and the suggestion in [Ref. 18] we set

$$\phi_{p_1} = f'_{p_1}\frac{v}{\|v\|}\,\|\dot{x}\|_1,\quad f'_{p_1} > 0 \tag{67}$$

Let λ denote the minimum eigenvalue of $\hat{A}^{-1}$. Then

$$v^T\hat{A}^{-1}\phi_{p_1} = f'_{p_1} v^T\hat{A}^{-1}\,\frac{v}{\|v\|}\,\|x\|_1 \ge \lambda\, f'_{p_1}\|v\|\,\|x\|_1 \tag{68}$$

Let us assume that there exists a constant $\mu_{p_1} > 0$ such that

$$\|h_{p_1}\| \le \mu_{p_1}\|\dot{x}\| \quad \text{for all } x \text{ and } \dot{x} \tag{69}$$

Then

$$|v^T h_{p_1}| \le \|v\|\,\|h_{p_1}\| \le \mu_{p_1}\|v\|\,\|\dot{x}\| \le \mu_p\|v\|\,\|\dot{x}\|_1 \tag{70}$$

From Eqs. (68) and (70) it follows that Eq. (66) holds if

$$\mu_{p_1} \|v\| \|\dot{x}\|_1 \leq \lambda f'_{p_1} \|v\| \|\dot{x}\|_1$$

or

$$f'_{p_1} \geq \frac{\mu_{p_1}}{\lambda} \tag{71}$$

In a similar manner the remaining inequalities in Eq. (65) are satisfied if we set

$$\phi_{p_2} = f'_{p_2} \frac{v}{\|v\|} \|x\|_1 \,, \phi_u = g'_u \frac{v}{\|v\|} \|u_m\|_1 \tag{72}$$

and choose f'_{p_2} and g'_u such that

$$f'_{p_2} \geq \frac{\mu_{p_2}}{\lambda}, \; g'_u \geq \frac{\mu_u}{\lambda} \tag{73}$$

where μ_{p_2} and μ_u are positive constants satisfying the inequalities:

$$\|h_{p_2}\| \leq \mu_{p_2} \|x\| \quad \text{for all } x \tag{74}$$

$$\|h_u\| \leq \mu_u \|u_m\| \text{ for all } x \text{ and } u_m \tag{75}$$

It should be noted that if h_{p_1}, h_{p_2} and h_u vanish then μ_{p_1}, μ_{p_2} and μ_u can be chosen zeroes. The positive design parameters f'_{p_1}, f'_{p_2} and g'_u are therefore unrestricted in this case.

To determine the matrix D, we need to solve the Liapunov Eq. (48) for P_2. The Jameson's method [Ref. 31] can be used to solve this equation numerically. However, if the matrices K_1, K_2 and Q are diagonal matrices P_2 can be found analytically and is given by:

$$P_2 = \left(\frac{1}{2} K_1^{-1} Q_1, \frac{1}{2} (Q_2 + K_1^{-1} Q_1) K_2^{-1}\right) \tag{76}$$

where Q_1, Q_2 are diagonal matrices such that $Q = \text{diag}\,(Q_1, Q_2)$.

We mention that the effectiveness of the integral adaptation, without the use of a reference model, has been investigated in [Ref. 17] for a manipulator with four degrees of freedom and that of the relay type (called unit vector) adaptation in [Ref. 18] for a manipulator with three degrees of freedom. However, the combined use of integral and unit vector adaptation has not been considered so far.

5. CONCLUSIONS

In this report we have reviewed some newer techniques (adaptive or non-adaptive) for the control of manipulators. The non-adaptive techniques are based on the inverse dynamics while those on adaptive control use the model reference approach and hyperstability

theory. The control laws suggested by these techniques contain some design parameters such as feedback gains. These parameters are restricted only by stability conditions but are free otherwise and can be chosen to meet other design criteria like, fast non oscillatory response, low position and velocity error in the prescence of disturbances, etc.

The implementation of these schemes is rather complex and is still in the experimental stage. At present, independent joint control in the form of PD or PID is used for each joint, with gains determined on the basis of some simple dynamical models or by linearizing around the set point. This type of control works well for slow motions and weakly coupled links. For faster motions and configurations with strong coupling, these local servos require very high-power actuators. For such motions, therefore, it becomes necessary to use some alternative schemes based on the ideas presented here.

There are some other control schemes that we have not considered. These include such schemes as force, and hybrid force/position control [Ref. 32-34] needed to control the end point forces. They can be derived using the same methodology as the position control. However, their study is even less mature than the study of position control and much remains to be done [Ref. 35] before they can be used in practice.

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SUMMARY/SOMMAIRE

Several control schemes based on inverse dynamics, and on model reference and hyperstability theory are presented.

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